

Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Paper 1

Practice paper – Set 1

Time allowed: 2 hours

Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

- 1** If $\sin \theta = \frac{3}{5}$, determine which **one** of the following statements **cannot** be true.

Circle your answer.

[1 mark]

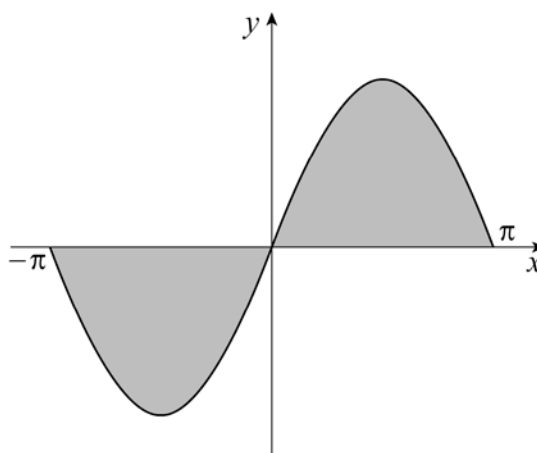
$$\sec \theta = -\frac{5}{4}$$

$$\cot \theta = -\frac{4}{3}$$

$$\cot \theta = \frac{4}{3}$$

$$\operatorname{cosec} \theta = -\frac{5}{3}$$

- 2** A section of the graph of $y = \sin x$ is shown.



Find the total shaded area.

Circle your answer.

[1 mark]

4

π

2

0

- 3 A straight line, l , has equation $4x - 7y = 11$

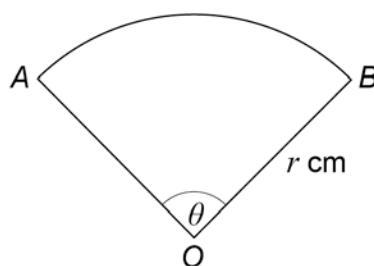
State the gradient of a line perpendicular to l .

Circle your answer.

[1 mark]

$\frac{7}{4}$ $-\frac{4}{7}$ $-\frac{7}{4}$ $\frac{7}{11}$

- 4 The diagram shows a sector AOB of a circle with centre O and radius r cm.



In sector AOB , $\theta = 1.5$ radians.

The sector has perimeter 10.5 cm.

- 4 (a) Find r .

[2 marks]

- 4 (b) Find the area of the sector.

[2 marks]

- 5 Anna says "If a and b are real numbers, then $a > b \Rightarrow a^2 > b^2$ "

Disprove Anna's statement.

[2 marks]

- 6 The n th term, u_n , of a sequence is given by $u_n = a \times 2^{-n}$ where $n \in \mathbb{N}$ and $a < 0$

Prove that this sequence is an increasing sequence.

[4 marks]

7 (a) $p(x) = 6x^3 - 19x^2 + 9x + 10$

Use the factor theorem to prove that $2x + 1$ is a factor of $p(x)$

[2 marks]

7 (b) Simplify the expression $\frac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$

[4 marks]

7 (c) State the values of x for which your answer to part (b) is **not** valid.

[2 marks]

8 A headteacher in a school of 1500 students noticed on 1 March 2017 that students were starting to bring fidget spinners into school.

She assumed that the number of students, N , who brought a fidget spinner into school would increase at a rate proportional to the product of N and the number of students who had not yet brought a fidget spinner into school.

8 (a) Construct a differential equation involving N , t and a positive constant, k , to model this situation, where t is the number of days after 1 March 2017.

[3 marks]

8 (b) Explain why the headteacher's assumption may not have been valid.

[1 mark]

9 A curve has equation $y = x^2 - 6kx + 9k^2$

Prove that the x -axis is a tangent to the curve for all values of k .

[4 marks]

-
- 10 (a)** Given that $u = 3^x$, write down an expression for $\frac{du}{dx}$
- [1 mark]**
- 10 (b)** Show that $\int_2^3 \frac{3^x}{3^x - 8} dx = \frac{\ln a}{\ln b}$ where a and b are prime numbers.
- [6 marks]**
- 11** A curve has equation $y = xe^{\frac{x}{2}}$
- Show that the curve has a single point of inflection and state the exact coordinates of this point of inflection.
- [8 marks]**
- 12** The functions f and g are defined by
- $$f(x) = 2^x, x \in \mathbb{R}$$
- $$g(x) = \sqrt{1-x}, x \in \mathbb{R}, x \leq a$$
- 12 (a)** State the maximum possible value of a .
- [1 mark]**
- 12 (b)** The function h is defined by $h(x) = gf(x)$
- 12 (b) (i)** Write down an expression for $h(x)$
- [1 mark]**
- 12 (b) (ii)** Using set notation, state the greatest possible domain of h .
- [2 marks]**
- 12 (b) (iii)** State the range of h .
- [2 marks]**
-

- 13** A spherical balloon is inflated so that its volume increases at a rate of 10 cm^3 per second.

Find the rate of increase of the balloon's surface area when its diameter is 8 cm.

For a sphere of radius r , surface area $= 4\pi r^2$ and volume $= \frac{4}{3}\pi r^3$

[6 marks]

- 14** Circle C_1 has equation $x^2 + y^2 - 8x - 14y = -40$

Circle C_2 has equation $(x - 16)^2 + (y - 12)^2 = 49$

- 14 (a)** Determine whether C_1 and C_2 intersect.

[7 marks]

- 14 (b)** Find the maximum distance between a point on C_1 and a point on C_2 .

[2 marks]

- 15 (a)** Prove the identity $\frac{\cos x}{\sec x + 1} + \frac{\cos x}{\sec x - 1} \equiv 2 \cot^2 x$

[3 marks]

- 15 (b)** Hence, solve the equation

$$\frac{\cos\left(2\theta + \frac{\pi}{3}\right)}{\sec\left(2\theta + \frac{\pi}{3}\right) + 1} = \cot\left(2\theta + \frac{\pi}{3}\right) - \frac{\cos\left(2\theta + \frac{\pi}{3}\right)}{\sec\left(2\theta + \frac{\pi}{3}\right) - 1}$$

in the interval $0 \leq \theta \leq 2\pi$, giving your values of θ to three significant figures where appropriate.

[5 marks]

- 16** A food packaging manufacturer makes cylindrical cans, each with volume 400 cm^3 .
 The cans are made from parts cut from sheets of two different metal alloys, A and B.
 The manufacturer cuts out circles from alloy A for the ends of the cans, and rectangles from alloy B, which are bent to make the sides of the cans.
 The cost per unit area of alloy A is 25% more than the cost per unit area of alloy B.
 Assume that the cost of wasted materials is negligible.

16 (a) (i) Using

C = total cost
 r = radius of can
 h = height of can
 p = price per unit area of alloy B

show that

$$C = \frac{5}{2}\pi r^2 p + \frac{800p}{r}$$

[3 marks]

- 16 (a) (ii)** Use calculus to find the radius of each can such that the cost of materials is a minimum.
 Fully justify your answer.

[5 marks]

- 16 (b)** State one further assumption you have made about the construction of the can in your modelling from part **(a)**.

[1 mark]

- 16 (c)** When the manufacturer starts mass production of the cans it is found that the cost per can is higher than the cost predicted by the model.

Explain a possible likely cause of the additional cost.

You should not do any further calculations.

[1 mark]

- 17** A tap is turned on to fill a water tank. At time t minutes after the tap is turned on, the depth of the water, x metres, increases according to the differential equation

$$\frac{dx}{dt} = (3 - x)t$$

The depth of the water is 0.65 metres when the tap is turned on.

Find the time taken for the depth of water to reach 1.85 metres.

[7 marks]

- 18** A student attempts to use proof by contradiction to prove that there are infinitely many primes.

The student's **incomplete** argument is shown below.

- Step 1: Assume the number of primes is finite, so that all the prime numbers can be listed in order: $p_1, p_2, p_3, \dots, p_n$
- Step 2: Construct a new number, q , by multiplying all the primes and adding 1
 $q = p_1 \times p_2 \times p_3 \times \dots \times p_n + 1$
- Step 3: q cannot be divided by any of the prime numbers p_1 to p_n so q must be a prime number not already in the list.
- Step 4: This contradicts the assumption made in step 1, so the number of primes cannot be finite. Therefore there are infinitely many primes.

Explain where the student's argument is incomplete.

[2 marks]

- 19 (a)** When θ is small, find an approximation for $\frac{\cos \theta - 1}{\theta}$

Simplify your answer.

[2 marks]

- 19 (b)** $f(x) = \cos x$

Using differentiation from first principles, prove that $f'(x) = -\sin x$

[6 marks]

END OF QUESTIONS

