PRACTICE PAPER
SET 1
CONDENSED PAPER

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level MATHEMATICS

Paper 2

Practice paper – Set 1

Time allowed: 2 hours

Materials
- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Section A

Answer **all** questions.

1 $y = \sqrt[3]{x}$

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

Circle your answer.

[1 mark]

$\dfrac{1}{3x^{\frac{2}{3}}}$ $\qquad$ $\dfrac{x^{\frac{2}{3}}}{3}$ $\qquad$ $3\sqrt[2]{x}$ $\qquad$ $\dfrac{3}{x^{\frac{2}{3}}}$

2 Find the number of terms in the series $\sum_{r=n}^{2n} r^3$, where n is a positive integer.

Circle your answer.

[1 mark]

$n-1$ $\qquad$ n $\qquad$ $n+1$ $\qquad$ $2n$

3 Find the exact value of $\displaystyle\int_1^3 \frac{2}{(x+1)(x+3)}\,\mathrm{d}x$

Fully justify your answer.

[6 marks]

4 (a) Show that the equation $\mathrm{e}^x - 3x = 0$ has a root, α, that lies between 1 and 2.

[2 marks]

4 (b) Use the Newton-Raphson method with $x_1 = 2$ to find:

4 (b) (i) x_2 and x_3 correct to 3 decimal places.

[2 marks]

4 (b) (ii) α correct to 3 decimal places.

[1 mark]

4 (c) Dan uses the Newton-Raphson method with $x_1 = 1$ to try to find α.

Describe what happens.

[2 marks]

5 The sum of the first 20 terms of an arithmetic sequence is 15.

The sum of the first 40 terms of this sequence is 250.

Find the number of terms of the sequence that are less than 100.

[7 marks]

6 (a) Write $\sqrt{3}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

[3 marks]

6 (b) A particle moves in a straight line so that its displacement, x metres from the origin, at time, t seconds, is given by

$$x = 8\cos\left(t + \frac{\pi}{6}\right) - 2\sqrt{3}\cos t\ ,\ t \geq 0$$

6 (b) (i) Show that $x = 2\sqrt{3}\cos t - 4\sin t$

[3 marks]

6 (b) (ii) Find the maximum displacement of the particle from the origin.

[1 mark]

6 (b) (iii) State the earliest time at which the maximum displacement occurs.

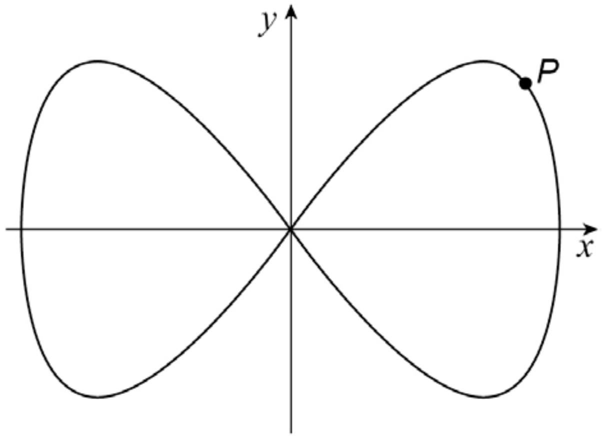
[1 mark]

7 The curve defined by the parametric equations

$$x = 2\cos\theta,\ \ y = 3\sin(2\theta) \text{ and } \theta \in [0, 2\pi]$$

is shown below.

The point $P\left(\sqrt{3}, \dfrac{3\sqrt{3}}{2}\right)$ is marked on the curve.



7 (a) Show that the equation of the normal to the curve at P can be written as $3y - x = \dfrac{7\sqrt{3}}{2}$

[8 marks]

7 (b) Show that the Cartesian equation of the curve may be written as $ay^2 + bx^4 + cx^2 = 0$ where a, b and c are integers to be found.

[4 marks]

8 A curve has equation $y^2 - 12xy + 3x^2 + 44 = 0$

Prove that the curve has exactly **two** stationary points, which lie on a straight line that passes through the origin.

[8 marks]

END OF SECTION A

Section B

Answer **all** questions.

9 Given vectors $\mathbf{p} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, find the value of $|\mathbf{p} - \mathbf{q}|$

Circle your answer.

[1 mark]

1 $\sqrt{13}$ $\sqrt{5}$ 5

10 The velocity, v, of a particle moving in a straight line is given by $v = \cos(3t)$ at time t.

State the acceleration of the particle at time t.

Circle your answer.

[1 mark]

$3\sin(3t)$ $-3\sin(3t)$ $-\frac{1}{3}\sin(3t)$ $\sin(3t)$

11 Two forces $\mathbf{F}_1$ newtons and $\mathbf{F}_2$ newtons act on a particle:

$$\mathbf{F}_1 = (3\mathbf{i} - 2\mathbf{j})$$
$$\mathbf{F}_2 = (2\mathbf{i} + 4\mathbf{j})$$

The unit vectors **i** and **j** are horizontal and vertical respectively.

The resultant of these two forces is **F** newtons.

11 (a) (i) Find **F**.

[1 mark]

11 (a) (ii) Find the acute angle that **F** makes with the horizontal, giving your answer to the nearest 0.1°.

[2 marks]

11 (b) A third force, $\mathbf{F}_3$ newtons, is applied to the particle.

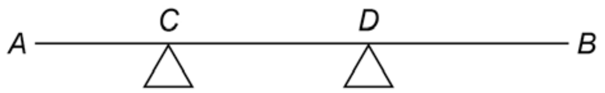
The resultant of the three forces is (–**i** + 3**j**) N.

Find $\mathbf{F}_3$.

[1 mark]

12 A uniform rod, *AB*, has length 4 metres and mass 6 kg.

It rests on two supports at *C* and *D*, where *AC* = 1 metre and *AD* = 2.5 metres.



12 (a) A particle of mass *m* kg is placed on the rod at a point *E*, 3.1 metres from *A*.

Given that the rod is on the point of tilting about *D*, calculate the value of *m*.

[3 marks]

12 (b) The particle of mass m is now removed and a particle of mass 2 kg is placed at a different point on the rod so that the rod is in equilibrium.

Given that the magnitude of the reaction at D is $\dfrac{13g}{2}$ newtons, find the distance of the 2 kg particle from A.

[4 marks]

13 A particle moves such that its velocity, $\mathbf{v}$ m s^{-1}, at time t seconds is given by

$$\mathbf{v} = \begin{pmatrix} 1-2t^2 \\ 2t \end{pmatrix}$$

When $t = 1$, the displacement of the particle from a fixed origin O is $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ metres.

Find the displacement from O when $t = 2$.

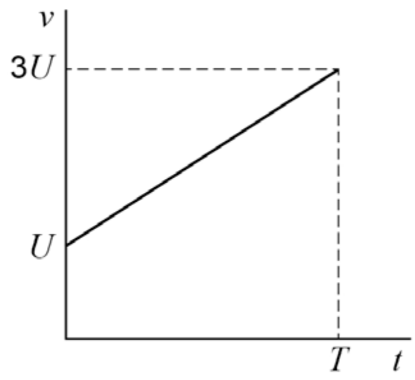
[5 marks]

14 A particle is initially at point O. It moves in a straight line with a constant acceleration of 0.68 m s^{-2}.

The initial velocity of the particle is U metres per second.

After T seconds, the particle has velocity $3U$ metres per second.

The velocity-time graph represents the motion of the particle.



14 (a) The particle travels 17 m in T seconds.

Find U and T.

[5 marks]

14 (b) Sketch a displacement-time graph for the motion of the particle.

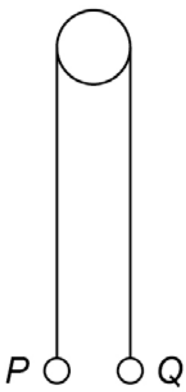
[1 mark]

15 **In this question use g = 9.8 m s^{-2}.**

Two particles P and Q, of masses 650 g and 850 g respectively, are attached to the ends of a light, inextensible string.

The string passes over a fixed peg. The particles are held at rest with the string taut.

The particles are then released.



15 (a) Find the magnitude of the acceleration of the particles and the tension in the string.

Fully justify your answer, stating any assumptions you make.

[5 marks]

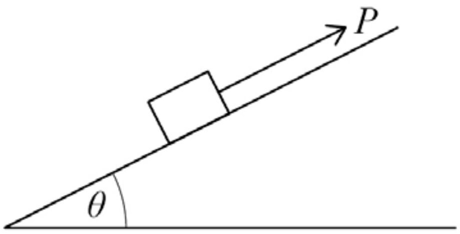
15 (b) Describe the effect of the assumption that the string is inextensible.

[1 mark]

16 The diagram shows a box of mass 2 kg on a rough plane inclined at $\theta°$ to the horizontal, where $\sin\theta = \frac{3}{5}$

The coefficient of friction between the box and the plane is $\frac{1}{3}$

The box is held in equilibrium by a force of magnitude P newtons acting up the plane as shown in the diagram.



16 (a) Given that the force is just sufficient to prevent the particle from sliding down the plane, show that $P = \frac{2g}{3}$

[5 marks]

16 (b) The force is now removed and the box slides down the plane.

Find the speed of the box when it has travelled one sixth of a metre down the plane, giving your answer in terms of g.

[4 marks]

17 Three points have position vectors $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 5 \\ \sqrt{3} \\ 7 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 9+4\sqrt{3} \\ 1 \\ 11+4\sqrt{3} \end{pmatrix}$

Prove that the points *A*, *B* and *C* are collinear.

[4 marks]

18 **In this question use *g* = 9.81 m s^{-2}.**

A golfer strikes a ball from a point, *A*, on the ground with speed 28.5 m s^{-1} at an angle of elevation 30º.

The ball travels towards a tree of height 9.5 m, which is at a horizontal distance of 45.4 m from *A*. The base of the tree is at the same horizontal level as *A*.

18 (a) Determine whether the ball will go over the tree.

Fully justify your answer.

[6 marks]

18 (b) Explain why your answer to part **(a)** might not be correct.

[1 mark]

END OF QUESTIONS