

Please write clearly, in block capitals.

Centre number

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Candidate number

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Surname

Forename(s)

Candidate signature

A-level MATHEMATICS

Paper 3

Practice paper – Set 1

Time allowed: 2 hours

Materials

- You must have the AQA Formulae for A-level Mathematics booklet.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions

- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 80.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Section A

Answer **all** questions.

- 1** If $\frac{dy}{dx} = \frac{1}{x}$ which **one** of these **cannot** be correct?

Circle your answer.

[1 mark]

$$y = \ln(5x+2) \qquad y = \ln(x) \qquad y = \ln(5x) \qquad y = \ln(5x) - 2$$

- 2** The equation $\sin\left(2x + \frac{\pi}{3}\right) = 0.3$ has four solutions in the interval $0 \leq x \leq 2\pi$

How many solutions does the equation $\sin\left(8x + \frac{\pi}{3}\right) = 0.3$ have in the interval $0 \leq x \leq 2\pi$?

Circle your answer.

[1 mark]

1 4 10 16

- 3 (a)** Find the first four terms of the binomial expansion of $\sqrt{16+x}$ in ascending powers of x .

[3 marks]

- 3 (b)** State the range of values of x for which the expansion is valid.

[1 mark]

- 4 Some experimental data for the variables x and y are shown in the table below.

x	1	2	3	4	5	6
y	5	8	15	26	47	85

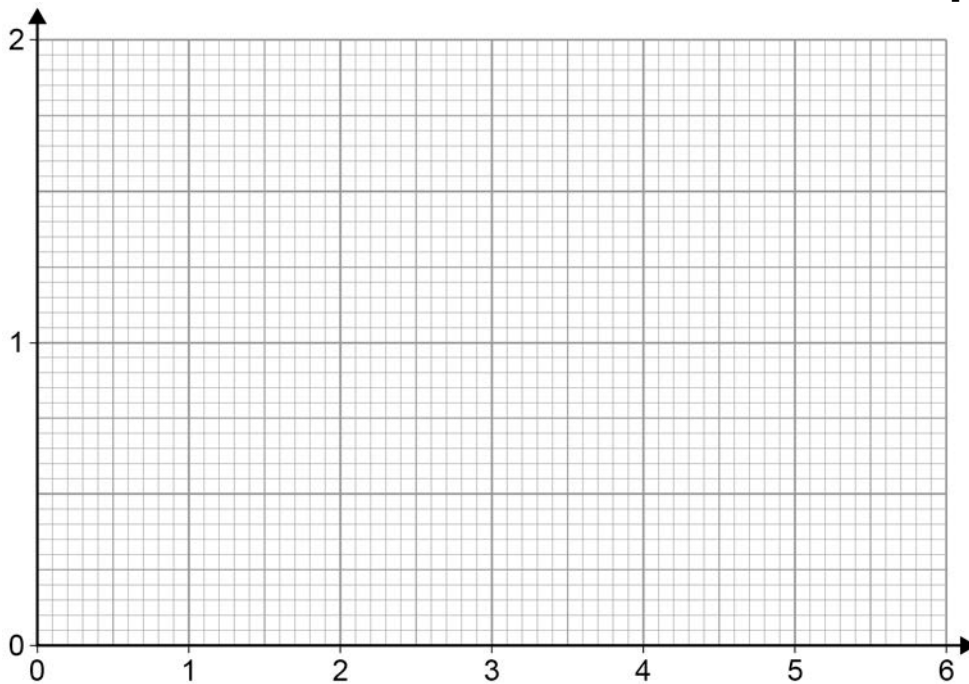
It is believed that there is a relationship between the variables that can be modelled by an equation of the form $y = ab^x$ where a and b are constants.

- 4 (a) Given that $y = ab^x$, show that there is a linear relationship between $\log_{10}y$ and x .

[4 marks]

- 4 (b) (i) Draw the graph of $\log_{10}y$ against x .

[2 marks]



- 4 (b) (ii) Hence estimate the value of a and the value of b .

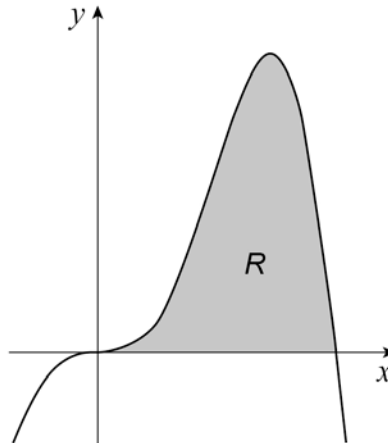
[3 marks]

- 4 (c) In a further trial of the experiment, when $x = 6.5$, $y = 92$.

Explain what this suggests about the accuracy of the model.

[2 marks]

- 5 Part of the graph of $y = x^2 \sin x$ is shown below.



Show that the area of the shaded region R is $\pi^2 - 4$

Fully justify your answer.

[9 marks]

- 6 The mass of a radioactive material decays exponentially.
It takes 10 years for the material to decay to 80% of its original mass.

Determine how many **more** years it takes for the material to decay to 10% of its original mass, giving your answer to 1 dp.

Fully justify your answer.

[6 marks]

- 7 The curve C_1 with equation $y = x^2$ is transformed onto the curve C_2 by a reflection in the x -axis followed by a translation of $\begin{pmatrix} 0 \\ 2a \end{pmatrix}$ where $a > 0$.

- 7 (a) On the axes below, sketch the curves C_1 and C_2

[3 marks]

- 7 (b) Show that the area of the region enclosed by C_1 and C_2 is $\frac{8}{3}a^{\frac{3}{2}}$

[6 marks]

- 7 (c) Given that the area enclosed by C_1 and C_2 is 72, find the value of a .

[2 marks]

8 A geometric sequence, S_1 , has first term a and common ratio r where $a \neq 0$ and $r \in (-1, 1)$

A new sequence, S_2 , is formed by squaring each term of S_1

8 (a) Given that the sum to infinity of S_2 is twice the sum to infinity of S_1 , show that $a = 2(1+r)$

Fully justify your answer.

[5 marks]

8 (b) Determine the set of possible values for a .

[2 marks]

**END OF SECTION A
TURN OVER FOR SECTION B**

Section B

Answer **all** questions.

- 9** Which of the following statements about the normal distribution is **incorrect**?
Tick one box.

[1 mark]

The median and the mean are the same

The curve is symmetrical about the mean

The curve has two inflection points

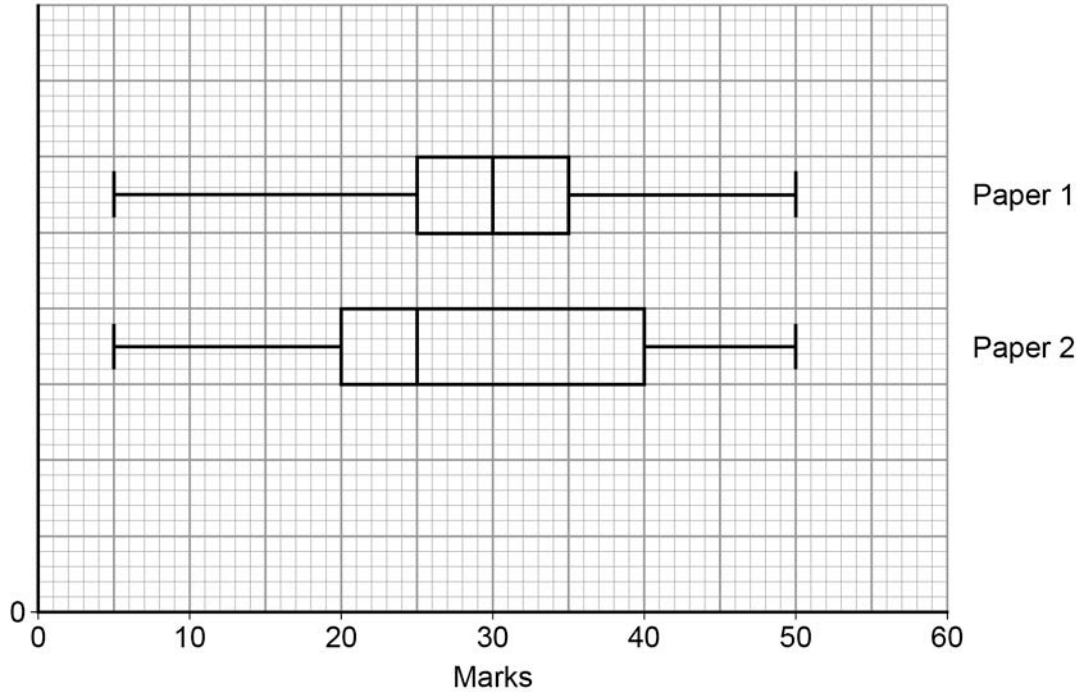
Approximately 95% of the values are within 3 standard deviations of the mean

10

A class of 40 students takes a mathematics test.

The test has two papers.

The students' results are represented by the box and whisker plots below.



Which **one** of the following statements is definitely **incorrect**?

Tick one box.

Each paper has a maximum possible mark of 50

The interquartile range of Paper 2 is twice the interquartile range of Paper 1

The data for Paper 2 is negatively skewed

The median of Paper 1 is five marks higher than the median of Paper 2

[1 mark]

- 11** Marta owns a gym with 350 members.
She has a complete list of all of the members and attempts to collect a simple random sample of size 20.

To collect the sample, Marta plans to assign three-digit random numbers to each member:

000 to 002 for member 1

003 to 005 for member 2

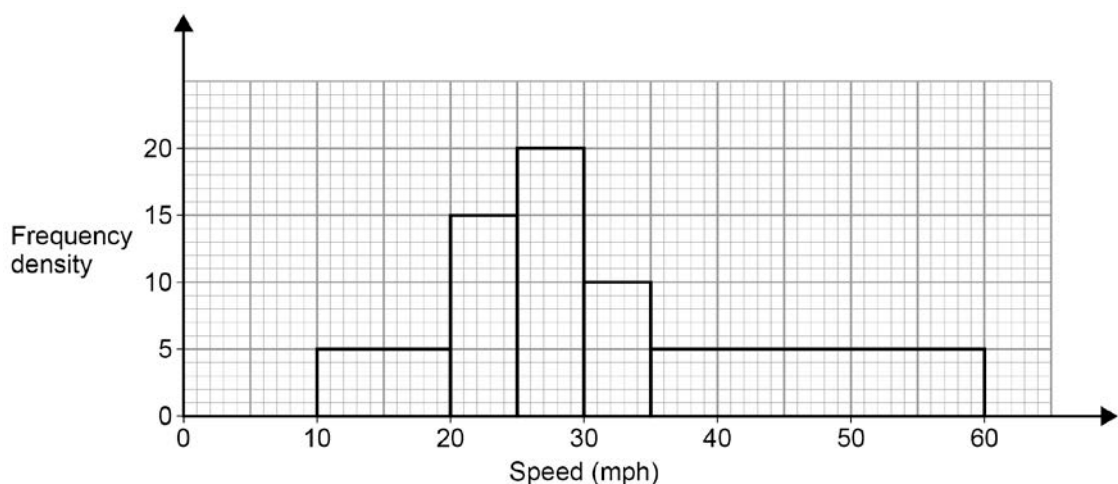
and so on.

She then plans to place 10 balls numbered 0 to 9 in a bag and draw three numbers, one at a time, without replacement, and select the appropriate member. For example, balls 1, 4 and 3 would generate the member allocated the digits 143.

The balls would then be replaced in the bag and the process repeated 19 times.

- 11 (a)** Give **two** criticisms of Marta's planned sampling method. **[2 marks]**
- 11 (b)** Suggest modifications that Marta could make to her sampling methodology that would generate an appropriate sample of size 20 for her survey. **[3 marks]**

- 12** The speed limit in a small village is 30 mph.
A resident is concerned that cars are travelling too fast through the village.
He recorded the speeds of the cars that passed him over a three-hour period and represented the data in the histogram below.



A car is chosen at random.

Find the probability that it was travelling at or over the speed limit.

[2 marks]

- 13** A supermarket promotion allows each customer to buy a maximum of four boxes of their premium own brand cat food 'Tiddles' while it is on special offer.

The probability distribution of X , the number of boxes of 'Tiddles' purchased per customer, is given by:

$$P(X = x) = \begin{cases} \left(\frac{2}{3}\right)^{x+2} + \left(\frac{1}{3}\right)^{x+3} & x = 1, 2, 3 \\ k & x = 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- 13 (a)** Find the exact value of the constant k .

[3 marks]

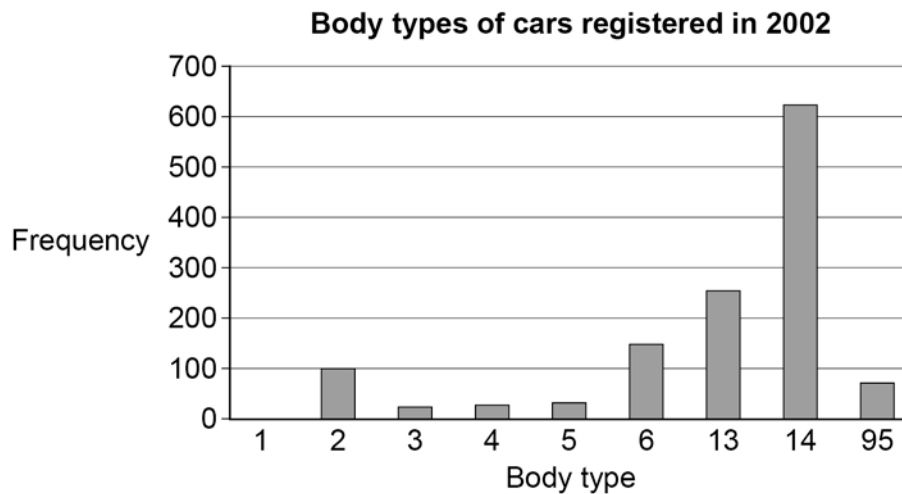
- 13 (b)** Find $P(3 \leq X \leq 4)$

[1 mark]

- 14 The bar chart (Figure 1) shows the distribution of the body types of cars as represented in the Large Data Set.

The data represents all 1299 cars in the data set which were registered in 2002.

Figure 1



Freya correctly identified the median numerical value from the bar chart as 14

Using your knowledge of the Large Data Set

- 14 (a) (i) State the body type corresponding to the numerical value of 14

[1 mark]

- 14 (a) (ii) Give one reason why Freya's median has no validity.

[1 mark]

- 14 (b) State one valid statistical measure for the data shown in the bar chart.

[1 mark]

Question 14 continues on the next page

- 14 (c)** The table below (**Figure 2**) summarises the data for all 1299 cars in the Large Data Set from 2002 by body type and region.

Figure 2

Body type	London	North West	South West	Totals
1	0	0	1	1
2	44	22	35	101
3	18	3	6	27
4	14	5	12	31
5	17	7	11	35
6	49	39	63	151
13	75	91	90	256
14	208	177	240	625
96	25	24	23	72
Totals	450	368	481	1299

Three different cars are selected at random.

Zac wants to work out the probability that one car is selected from the North West and two from the South West.

This is his working out:

$$\begin{aligned}
 P(\text{NW, SW, SW}) &= \frac{368}{1299} \times \frac{481}{1299} \times \frac{481}{1299} \\
 &= 0.0388
 \end{aligned}$$

Explain the errors Zac has made in his working.

[3 marks]

-
- 15** The time, T , spent on the internet by 16 to 24-year-olds in hours per week is modelled by a normal distribution with mean 30 and variance 64.
- 15 (a) (i)** Find $P(T > 40)$ **[1 mark]**
- 15 (a) (ii)** Find $P(28 < T < 48)$ **[1 mark]**
- 15 (b)** Explain why $P(T = 30) = 0$ **[2 marks]**
- 15 (c)** 8% of 16 to 24-year-olds spend more than y hours on the internet.
Find the value of y correct to 1 decimal place. **[1 mark]**
- 15 (d)** Find $P(T < 0)$ and hence comment, in context, on the likely validity of the model for T . **[2 marks]**
- 15 (e)** Nine 16 to 24-year-olds are chosen at random and the number who spend time on the internet within one standard deviation of the mean is modelled by the random variable X .
Assuming independence, find $P(X \geq 5)$ **[4 marks]**
- 16** A social media app was rated 'excellent' by 70% of its users based upon a large sample. At the start of 2017, the owners of the app introduced some new features, and received mixed feedback from their users.
They wondered if the percentage of users rating the app as 'excellent' had changed. A random sample of 28 customers were asked to rate the app and 15 rated it as 'excellent'.
- 16 (a)** Use a binomial distribution to investigate, at the 5% level of significance, whether there is evidence to support the claim that the percentage of users who rate the app as 'excellent' has changed. **[5 marks]**
-

- 16 (b)** Derek, one of the owners of the app, was against the introduction of the new features. He claims the percentage who rate the app as 'excellent' has dropped to under 55%.

Comment on whether Derek is correct.

[2 marks]

- 17** Ellie eats regularly at a Mexican restaurant.
Let A be the event 'Ellie orders nachos as a starter'.
Let B be the event 'Ellie orders a chicken burrito as a main course'.

$$P(B|A) = \frac{9}{13}$$

$$P(A \cap B) = \frac{9}{20}$$

$$P(A' \cap B) = \frac{1}{4}$$

- 17 (a) (i)** State, in the context of the question, what the event $A' \cap B$ represents.

[1 mark]

- 17 (a) (ii)** State in terms of A and B the event that Ellie orders nachos **or** a chicken burrito.

[1 mark]

- 17 (b) (i)** Find $P(A)$

[1 mark]

- 17 (b) (ii)** Find $P(A|B)$

[3 marks]

- 17 (c)** Determine whether events A and B are independent.

[1 mark]

- 18** In a 65 g bar of high protein chocolate, the sugar content can be modelled by a normal distribution with mean 8 g and variance 0.2 g^2 .
- There is a change in the manufacturing process, and the makers are concerned that this may have caused the mean sugar content of the bar to increase.
- A random sample of 60 bars is selected and the sugar content, x grams, of each bar is measured.
- For this sample $\sum x = 492.3$
- Test at the 1% significance level whether the mean sugar content of a bar has increased.

[6 marks]

END OF QUESTIONS